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The principal objectives of this research project were (1) improvement of algorithms for solving large-scale, block-structured convex programming problems, (2) approximation of optimization problems for purposes of computational solution, and (3) improvement of algorithms for nonsmooth optimization, as well as (4) use of the research results to improve problem-solving ability in application areas. Progress was achieved in all of these areas, and was documented in eight papers prepared for journal publication as well as one Ph.D. dissertation and two technical reports.

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Final Technical Report

Computation and Theory in Large-Scale Optimization

Grant No. AFOSR-91-0089

University of Wisconsin-Madison

1. Abstract

The principal objectives of this research project were (1) improvement of algorithms for solving large-scale, block-structured convex programming problems, (2) approximation of optimization problems for purposes of computational solution, and (3) improvement of algorithms for nonsmooth optimization, as well as (4) use of the research results to improve problem-solving ability in application areas. Progress was achieved in all of these areas, and was documented in eight papers prepared for journal publication as well as one Ph.D. dissertation and two technical reports.

2. Overview of Research Accomplished

This overview is organized according to the major categories in the abstract above.

1. Improvement of algorithms for large-scale optimization.

The main effort in this area was devoted to improvement of methods for solving so-called *scenario analysis* problems: that is, problems of (possibly multistage) stochastic optimization in which the uncertainties are finitely distributed. Work under this project succeeded in demonstrating that a decomposition technique based on the bundle method for solving nonsmooth optimization problems was effective in solving loosely-coupled scenario analysis problems. Computational comparisons showed that for such problems the bundle decomposition method performed much faster (up to a factor of 1,500) than the progressive hedging algorithm (the other currently-proposed candidate for solving scenario analysis problems). This work on scenario analysis is documented in the dissertation [D1], the journal paper [P8], and the technical reports [R1], [R2], from which further information can be obtained.

2. Approximation of optimization problems.

Basic research on approximation of *normal maps*, a kind of nonsmooth function that can be used in equations to model necessary conditions for optimization or equilibrium problems, is contained in [P1], [P2], and [P5]. The paper [P1] develops homeomorphism conditions that are sufficient to permit a classical embedding method to be used to solve such equations. This method initially uses very simple functions as approximations, then successively deforms the approximations into the functions actually appearing the problem. A more sophisticated (homotopy) method that can be used when these conditions do not hold is currently under development.

[P2] contains necessary and sufficient conditions for application to normal maps of the implicit-function theorem previously developed by the principal investigator. This theorem provides the foundation for local approximation of such functions, including computational estimates. [P5] shows how to obtain the results of [P2] in a much simpler way if a certain derivative matrix appearing in the problem is symmetric. This symmetry occurs, in particular, in those normal maps arising from optimization problems over polyhedral convex sets.

3. Improvement of algorithms for nonsmooth optimization.

The main progress in this area was the further development of a Newton method for solving nonsmooth equations, including those obtained from the necessary conditions for optimization or equilibrium problems. This method approximates the nonsmooth equation being solved by a local model generated using a *point-based approximation*. This local model, although generally also nonsmooth, is in many practical cases easy to solve. By solving a sequence of such problems one approximates the solution of the original nonsmooth equation. This work is documented in [P7]. As its basic technique uses approximation, it also contributes to objective (2).

4. Improving problem-solving in applications areas.

[P3], [P4], and [P6] applied convex analysis to solution techniques for certain applied problems. [P3] and [P4] dealt with military force-modeling problems in which it is important to estimate the relative contribution of different systems on the battlefield. The results reported there show that the so-called "eigenvalue weights," which have appeared in force-modeling research since the late 1960s, are actually marginal values, and therefore that they can be used to estimate relative contributions of different systems. [P6] applies a similar approach to the problem of cost/benefit analysis with incommensurable attributes. It shows that, in particular, the importance weights computed by the popular Analytic Hierarchy Process measure the effect of attribute tradeoffs on a benefit/cost ratio. This provides an explanation of the function of these weights, where previously none had been available.

3. Results from Research Activity

The following scientific works acknowledge support from Grant AFOSR-91-0089.

a. Dissertation.

[D1] B.J. Chun, *Scenario Analysis Modeling and Decomposition Methods for Optimization Under Uncertainty*, Ph.D. Dissertation, Department of Industrial Engineering, University of Wisconsin–Madison, August 1992.

b. Papers.

[P1] S.M. Robinson, "Homeomorphism conditions for normal maps of polyhedra," in: A. Ioffe, M. Marcus, and S. Reich, eds., asl Optimization and Nonlinear Analysis, Longman (Pitman Research Notes in Mathematics Series, No. 244), Harlow, Essex, England (1992) 240–248.

[P2] S.M. Robinson, "Normal maps induced by linear transformations," *Mathematics of Operations Research* **17** (1992) 691–714.

[P3] S.M. Robinson, "Shadow prices for measures of effectiveness, I: Linear model," accepted by *Operations Research*.

[P4] S.M. Robinson, "Shadow prices for measures of effectiveness, II: General model," accepted by *Operations Research*.

[P5] S.M. Robinson. "Nonsingularity and symmetry for linear normal maps," accepted by *Mathematical Programming*.

[P6] S.M. Robinson. "Minimax cost/benefit analysis," Manuscript, August 1992, submitted to *Management Science*.

[P7] S.M. Robinson. "Newton's method for a class of nonsmooth functions," revised version of November 1992, submitted to *Set-Valued Analysis*.

[P8] B.J. Chun and S.M. Robinson. "Scenario analysis via bundle decomposition," Manuscript, December 1992, submitted to *Annals of Operations Research*.

C. Technical Reports

[R1] B.J. Chun, S.J. Lee, and S.M. Robinson. "An implementation of the bundle decomposition algorithm," *Technical Report No. 91-6*, Department of Industrial Engineering, University of Wisconsin–Madison, August 1991.

[R2] B.J. Chun, "Scenario analysis modeling and decomposition methods," *Technical Report No. 92-8*, Department of Industrial Engineering, University of Wisconsin–Madison, 1992.

4. Participating Professionals

The following professional personnel received salary support from Grant AFOSR-91-0089.

- Bock Jin Chun, Research Assistant.
- Mohamed N. Azaïez, Research Assistant.
- Sang Jin Lee, Research Assistant.
- Laura Morley, Research Assistant.
- Yonca A. Özge, Research Assistant.
- Stephen M. Robinson, Professor.
- Hichem Sellami, Research Assistant.

5. Degrees Awarded

B.J. Chun received the degree of Doctor of Philosophy (Industrial Engineering) in August 1992. Among the other participants, M.N. Azaïez, S.J. Lee, Y.A. Özge, and H. Sellami are continuing students in the doctoral program (in Industrial Engineering except for S. J. Lee, who is in Business), and are expected to receive degrees in the future.

6. Inventions and Patent Disclosures

During the work under this grant, there were no inventions that appeared to have any patent possibilities. Other (non-patentable) discoveries are contained in the papers reported above.

7. Other Information

Further information about any of the activities reported above, or other aspects of this research program, can be obtained from the principal investigator, Stephen M. Robinson, at the Department of Industrial Engineering, University of Wisconsin–Madison, 1513 University Avenue, Madison, WI 53706-1572, telephone (608) 263-6862, fax (608) 262-8454, email smr@cs.wisc.edu.